Articles

# "Logic of children" and "Logic of subject matters": Effect of an instructional intervention on understanding ratio concepts based upon children's informal knowledge [1]

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The paper tested the hypothesis that an instructional intervention based on children's informal knowledge acquired through everyday life would improve their ability in mathematical problem solving. The study deals with the ratio concept which is extremely difficult for students to understand. According to our previous studies, Japanese children have relevant informal knowledge before the formal teaching of the ratio concept, namely they acquired some basic meaning of the notion, and they are informally able to solve problems relating to comparing quantities that involve percentage. A new curriculum integrating such informal knowledge was developed and implemented in a fifth-grade class of a Japanese elementary school. The results demonstrated that ability in problem solving improved highly in students who were immersed in the instructional intervention compared to those who followed the traditional textbook program.

**Key words** : informal knowledge, ratio, instructional intervention, school knowledge

It has been criticized that sujbect matters taught in schools like mathematics or science do not necessiarly reflect knowledge which children acquired through everyday life. In Japan, for example, some investigators distinguished such disconnection as "school knowledge" and "everyday knowledge." (Saeki, et al., 1992). Verschaffel, De Corte, & Lasure (1994) gave scientific evidences about this claim through a investigation using word problems. They demonstrated that children have a strong tendency to exclude real-world knowledge in solving problematic word problems. This tendency was repeatedly shown in many studies (Reusser & Stebler, 1997; Verschaffel, L., De Corte, E., & Lasure, S., 1999; Yoshida et al., 1997).

While there were many factors to produce such disconnection between mathematics learning in schools and real-world knowledge, we assumed that one of factors is current curriculum was constructed based on logic of principles of academic disiplines such as mathematics or science. We call this type of curriculum as oen with "logic of subject matters." Investigations suggested that this

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type of curriculum impoverished problem solving of children (De Corte, et al., 1996; Stevenson & Bartsch, 1992).

In light of recent many investigations, what lacked in curriculum based on the logic of subject matters are many findings on processes of problem solving, on strategies used in solving problems, or informa knowledge which children acquired through everyday life (Greeno et al., 1996; De Corte et al., 1996). We call curriculum in which contained research-based results as one with "logic of children" (Yoshida, 1999). While instructional interventional studies were recommented that constructed new curriculum based on cognitive researches, intervened in classrooms, and assessed effect of such intervention (Carpenter et al., 1993), there were few investigations relating curriculum to learning and instruction. Thus, it has been expected to develop an instructional intervention theory based on both logic of children and subject matters. Recently, the holistic theory was proposed that include such basic characteristics as the teaching-learning environment, reciprocal communication with practioners, and induction of a fundamental change of teacher's belief systems (De Corte, 2000).

The present study constructs a new curriculum based on logic of children, give an experimental intervention to children, and assess a effect of such intervention. In doing so, the study adopt a holitic approach involving some factors as suggested in De Corte (2000). Thus, the purpose of the present study was to confirm the hypothesis that experimental intervention based on logic of children would promote understanding of concepts more compared to the traditional approach based on logic of subject matters.

In the present study, the following three factors were created in instructional intervention. The first was to construct the new curriculum which was combined traditional contents with informal knowledge in children. A concept dealt with the present study is percentage which is one of concepts taught as ratio in Japan. There were very few researches about percentage in viewpoint of cognitive psychology although many ones based on behaviorism (Parker & Leinhardt, 1995). However, recently Kawano & Yoshida (1999) reported that students acquired such meaning as quantity in % or meaning as part-whole relation in % without learning it formally. In addition, they were able to solve ratio problems of the second term "compare quantity = base one × percentage" in % by using informal knowledge (Yoshida et al., 2000).

The new curriculum was constructed in the following two frameworks based on these research-based results: The first was to stress meaning as quantity. In Japanese textbooks, concept as symbol or equation was emphasized in solving ratio problems. The only way of solving ratio problems is to apply eauations to given problems (Keirinkan Publisher, 2000). However, the new curriculum introduces aspect of quantity in ratio concepts, which children already acquired such aspect to some extent before learning formally. The second framework was to change sequence of contents. The national guildliens on ratio require to teach first the first term "ratio including percentage = compare quantity÷base one", then the second term , and then the third term "base quantity = percentage ÷ compare one". However, in the new curriculum the second term was taught first, then the first term, and finally the third one.

The second factor involved in the present study is an aspect of meta-cognition. The previous investigators tried to train directly meta-cognitive skills in students (Campione et al., 1989). In the present study, however, we aimed to develop such skills as estimating magnitude of an answer in problems by utilizing a material devised to represent the magnitude of the answer.

Third, the view of learning as a social process is central in productive learning (Brown et al., 1989). The conception of learning as collaborative process was also taken into account in the present study. In fact, we introduced small group activities among students as many as possible in addition to active interaction between teacher and students.

## Method

### Participants

The experimental group consisted of 35 fifth graders from one of the three classes in a public elementary school, and the textbook group consisted of 71 fifth graders from the remaining classes in the same school. The school was located in a middle-class suburb of a medium-sized city in Japan.

### A framework of the experimental curriculum

Before describing the framework, it is necessary to explain national curriculum on ratio concepts involved percentage. In Japan, percentage is taught as part of ratio concepts in fifth grade of elementary school. The most important concept in ratio on the curriculum is an equation of ratio = compare quantity ÷ base one. This is called as the first term in ratio. The second term in ratio is compare quantity = base one × percentage set by changing the first term. The third term is base quantity = compare one ÷ percentage set by changing also first term. These three terms are introduced in context of both decimal number times and percentage. Almost of textbooks stress that ratio problems could be solved by applying these three terms on real problems (for example, Keirinkan, 2000). Number of lessons in the unit for ratio including percentage and decimal number times is 13 lessons. However, lessons for instructional intervention in the present study was 8 ones, in which three terms in ratio were taught.

The first element of the framework is that curriculum composed in this study is based on both formal knowledge defined in textbook and informal knowledge which students acquired. Previous investigations found that children already knew basic meaning of part-whole in ratio (Kawano & Yoshida, 1999). Then, based on such result, percentage was introduced not in terms of the equation but in viewpoint of part-whole relationship. Further, based on informal rich knowledge on second term in ratio (Yoshida et al., 2000), sequence of presenting equations was changed from 2nd

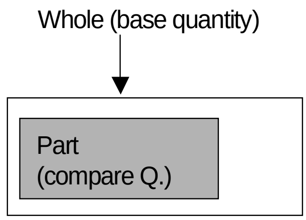


**Figure 1. A ratio model used in the intervention**

**Table 1. Contents of each lessons in the two groups.**

| | Experimental group | Textbook group |
|---|---|---|
| 1 | Students learn how to read and write %, and do base quantity, compare one, and ratio. They also are taught a ratio model. | Teacher shows table with the number limit and candidates's number to each club, then introduces basic meaning of ratio. |
| 2 | Students know how to use the ratio model in representing a given ratio problem. They learn to use the model in representing both of two quantities in problems as base quantity. | Teacher introduced an equation of the first term by using decimal number times. Students learn how to solve problems by using the equation. |
| 3 | Students represent a given problem by the model, estimate rough magnitude of %, and compare magnitude of problems's answers by using such rough magnitude of %. | Students learn how to solve ratio problems of the second term by using decimal number times. |
| 4 | Students learn relation between % and decimal fraction times by representing problems as the ratio model. | Students learn how to solve ratio problems of the third term by using decimal number times. |
| 5 | Students learn how to solve ratio problems of the second term. | Students learn relation between decimal fraction times and %. |
| 6 | Students learn how to solve ratio problems of the first term. | By using % students learn how to solve ratio problems of both the first and second terms. |
| 7 | Students learn how to solve ratio problems of the third term. | Students learn how to solve ratio problems of the third term on the context of %. |
| 8 | Drill practice and summary of this unit. | Drill practice and summary of this unit. |

term, to 1st, and then 3rd term although order in textbook was from 1st term, to 2nd, and 3rd one.

Concerning to the second element, we devised basic subject material for representing magnitude of %. By using it frequently, we postulated that students would estimate magnitude of answer in problems before computing and then control their own activities in solving problems. The material is shown in Figure 1. We call it ratio model. An outer frame of this figure indicates base quantity or a whole in problems, and an inner one does compare quantity or a part in problems. The inner frame moves freely inside or to outside of the outer one, following to requirement of problems. It is expected that students would be able to estimate magnitude of percentage.

### Experimental intervention

Table 1 shows outline of each lesson in both the Experimental and Textbook groups. As suggested from Table 1, first three lessons in the Experimental group teach basic meaning of percentage or ratio by utilizing the ratio model. In the first lesson, a teacher explained how to read and write percentage, taught meanings in base quantity, compare one and ratio, and then explained the ratio model. It was instructed in the second lesson to represent relation between base and compare quantities by utilizing the model. The teacher showed it was possible for students to represent magnitude of answer in problems by operating the ratio model. In the third lesson, he taught students were able to estimate rough magnitude of answer in problems by using the model.

Contents of the fourth lesson in Experimental group were same to ones in Textbook classes. Main contents in the lesson were that percentage is changed to decimal number or vice versa and ratio is shown in both percentage and decimal number times.

Equations of the three terms in ratio were taught from fifth to seventh lessons ratio in the Experimental group. Sequence of teaching was from second, to first and to third terms, unlike order of the Textbook group.

Throughout these lessons the teacher introduced small group activities actively as well as direct instcution or active interaction between teacher and students. The teacher gave a goal for group activity in introducing it and asked students to attain the goal by themselves. For example, students were required to solve ratio problems in each group by using the ratio model.

The all eight lessons in the Experimental group were filmed.

### Lessons of the Textbook classes

Two classes in the textbook group followed mathematics textbook. Main contents in each lesson are shown in Table 1. Throughout the lessons teachers stressed that students were able to solve ratio problems by applying ratios' equations on real problems. Teachers taught the first term of ratio, gave problems, and asked them by using the equation. They did similary about both the second and third terms of ratio. They used typical subject materials shown on the textbook to represent relation between base and compare quantities. None of them instructed such strategies as estimation in solving ratio problems. There were small group activities in the Textbook group. All lessons in one of the classes was filmed. However, because of limitation in number of investigators, lessons in another class were not filmed.

### Tests

Pre-test. Informal knowledge on percentage was tested in the pre-test. Main categories in the test were basic meanings on percentage (5 problems), magnitude of percentage (4 ones), and second term on ratio (3 ones). Some of these test items are shown in Appendix 1. The pre-test was conducted collectively before starting a ratio unit.

Post-test. There were main four categories in the post-test; word problems on ratio (2 problems each from three terms in ratio), transforming problems (5 ones), relational judgement (2 ones), and estimation (2 problems each from three terms in ratio). Some of test problems are also shown in Appendix 2. The post-test was administrated collectively 10 days after finishing lessons of the ratio unit.

## Results

### Simple description during lessons

Outline of lesson in both groups were explained in Method section. However, such outline might little gave a flavour of real lessons to readers. Because such flavour would be manifest in social interaction especially for the Experimental group, we present protocols during lessons. The following was excerption from the sixth lesson in which teacher gave students problem and asked them to solve it by using the ratio model in small group.

S1 : (read the problem. There is 35 students in Makoto's class. 26 out of them were born on November. How many percentage of students born on November were there?)

S2 : Let's use the ratio model!

S1 : Where is the number of students in class?

S3 : This is the number of all students in class (pointing outer frame of the model) because we have to find ratio of students born on November.

S4 : OK, this is students born on November (pointing inner frame of the model).

S2 : I'll try. As students' number of the class is 35 and students born on November is 26, probably this extent (moving the inner frame).

S1 : A little bit larger, (S2 moves the inner frame again), that's it.

S3 : It probably 75%, isn't it?

S4 : Ya, it is so.

S1 : Then, how to compute, is it 26÷35?

S2 : (compute on a sheet of paper), 0.7428, it is strange, I can't divide clearly.

S3 : I'll try again. (compute), 0.7428, well the answer was same. The answer was correct, I think.

S1 : The answer was about 74%, wait, wait, ····, Oh, I remembered the answer in the ratio model was about 75%. Both answers were same. That is OK.

S4 : Yes, I also forgot the ratio model. The answer is OK.

We present similar lesson situation with same problem in the Textbook group to give real classroom presence. However, the situation was observed in interaction between teacher and students. The following was excerption from the protocol to indicate interaction between teacher and students.

T : (The teacher is waiting responses from students after giving the problem same to one used in the Experimental group)

S1 : (spontaneously), teacher, I didn't understand.

T : Didn't you do?

S1 : Teacher, I can't divide it cleary.

S2 : Teacher, I can't also do it.

S3 : Teacher, me too.

T : OK, let's it think all together!

After these interaction with students, the teacher required all students to read aloud the problem and wrote down the equation on the blackboard. She explained again what base and compare quantities are in the equation and applied it to the problem. Then, she asked to compute it.

S1 : Teacher, I couldn't again divide it.

T : Although you were not able to do it, did you understand my explanation?

S3 : As you taught, I calculated. However, I couldn't divide it. It is a little bit strange.

Students seemed not to be convinced of the result of problem solving, as far as we observed the lesson. They behaved as if they had belief that problems given in their classroom should be divided clearly. This tendency was fairly different from one observed in the Experimental group.

Teachers in the Textbook group utilized sometimes small group activity in their lessons. The teacher we observed did such activity at least once a lesson. Thus, it seemed that there was less basic difference between the Experimental and Textbook groups in collaborative activity during lessons.

**Assessment in both the pre- and post-test**

Performances in the pre-test. Figure 2 indicates percent corrects for meaning of percentage, representation of magnitudes, and the second term in the pre-test. There were no statistical differences in these three categories between the Experimental and Textbook groups.

Performances in the post-test. Results of correct percentage in word problems are shown in Figure 3. There were significant differences between both groups in the first term, t(104)=7.681, $p$<.01, in the second one, t(104)=4.036, $p$<.01, in the third one,

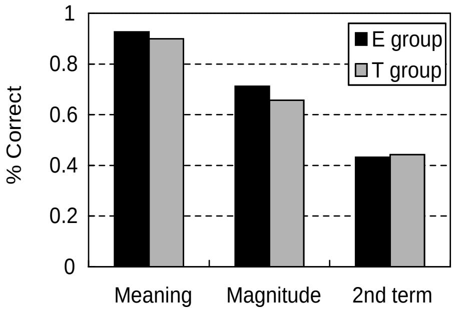


**Fig. 2. Percent Correct for each category in the pre-test**

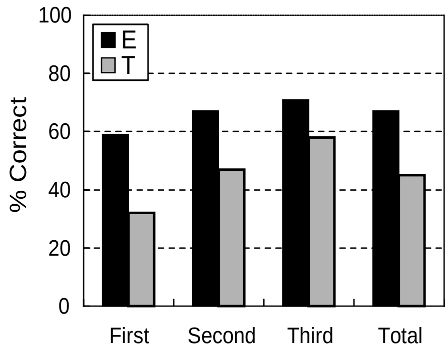


**Fig. 3. Correct percentages in ratio word problems**

t(104)=2.043, $p$<.025, and in total, t(104)=3.918, $p$<.01. These results cleary indicate superior ability of problem solving on ratio in the Experimental group to the Textbook one.

In solving ratio problems students were asked to explain how to solve problems for each problem in such way as teacher could understand. We divided these explanations into main three strategies; calculation, estimation, and no response. The calcultion strategy involved ones to explain just computational process or to describe application of the equation. The estimation strategy involved like selecting an operation by estimating magnitudes of an answer, computing after drawing the ratio model, or re-computing by comparing the magnitude of answer which student estimated during solving with the obtained answer.

Figure 4 shows percentages of each strategy utilized in solving problems for the two groups. As indicated in the Figure 4, main strategy in solving ratio problems was calculation while the Textbook group used it in solving problems more than the Experimental group, t(104)=2.118, $p$<.025. On the contrary, although students in the Textbook group little used the the estimation strategy, the

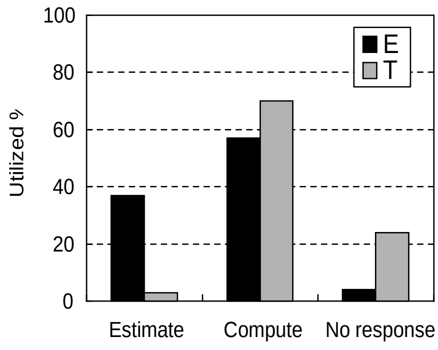


**Fig.4 Percentages of strategies utilized in solving ratio word problems**

Experimental group cleary relied upon the strategy, t(104)=9.583, $p$<.001. Higher correct percentages of problem solving in the Experimental group would be partly due to frequent utilization of the estimation strategy.

Correct percentages in transformation, relational judgement, and estimation tasks are shown in Figure 5. While there was no significant difference between the two groups in the transformation task, the Experimental group indicated significantly superior performances to the Textbook group in the relational judgement task, t(104)=10.043, $p$<.001, and in the estimation one, t(104)=3.569, $p$<.01. The relational judgement task required students to judge order of magnitude among three persons based on their relationship only. So, it is usually hard task for students to solve the relational judgement task as suggested in fairly low correct percentage (32%) of the Textbook group. However, the Experimental group scored 2.7 times more than the Textbook group in the task.

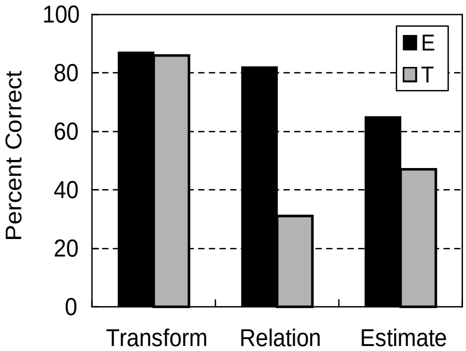


**Fig. 5 Correct percentages in ratio problems**

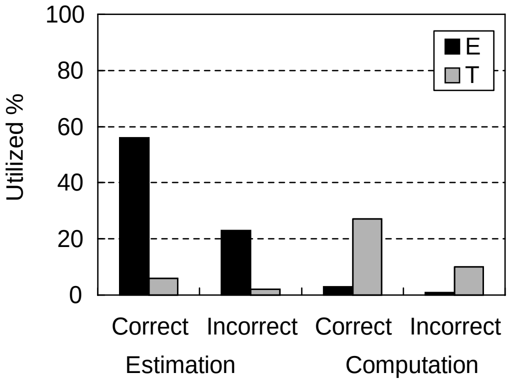


**Fig. 6 Percentages of strategies utilized in solving the estimation problem**

In the estimation task students were asked to select one of answers by using figure drawing or other ways without computing problems, and to write down the reason why they selected the answer in addition. Thus, many students answered these problems by estimating answers as the task required. But, there were some students who answered by calculating these problems directly. So, at first we divided answers of problems into correct or not, and students' strategies were divided into either the estimation or calculation in these problems. Other strategies like non-explanation or hard one to identify were excluded in this analysis. Figure 6 shows percentages which these two strategies were utilized for both answers in each group. As suggested in Figure 6, students in the Experimental group highly depended upon the estimation strategy. On the contrary, the Textbook group did upon the calcuation strategy although it was instructed not to use one. Thus, it was quietly clear that the Textbook group was not able to utilize sophisticated strategy such as estimation.

## Discussion

The present study aimed to test effect of the experimental intstructional intervention based on a holistic theory on understanding mathematical concepts like ratio. We included four main elements in this investigation; (1)The experimental instruction was built on informal knowledge on ratio, (2)The present intervention intended to foster meta-cognitive ability to judge estimate of answer in problems by intrdocuing the new material which represented whole and part relation in ratio, (3)We utilized in the lessons small group activity to activate interaction among students, (4)Teacher tried to possess positive attitude and belief related to mathematics.

However, in fact, it seemed that the exerimental group did not necessary differ from the Textbook group in all four elements. For example, teachers in the Textbook group utilized fairly small group discussion to activate interaction among students and as far as we observed lessons in both groups, we didn't feel big difference in frequency of such activity between the two groups. In addition, teachers in the Textbook group did not know recent investigations on student's thinking or strategies in mathematical problem solving and seemed to have traditional belief to mathematics. However, they, in general, put a great value for fostering positive mathematical attitude and did less value for direct teaching on mathematical concepts. In fact, they sometimes introduced small group activity to elicit spontaneous responses from students, and introduced incorrect answers from students to lessons and discussed why such answers were incorrect in whole class. Thus, it appeared that there were few basic differences in third and fourth elements between the two groups.

However, the Experimental group highly differed from the Textbook one in the first and second elements.The textbook group did not include informal knowledge in their teaching, utilize the material which represented magnitude of percentage visually, and instruct estimation strategy at all.

As been evident from the results in the post-test, the experimental group showed superior ability on solving ratio problems to the Textbook group and, in addition, indicated correct performance 2.7 times more compared to one of the Textbook group in hard problems required relationship among three peoples without qualifying. These successful performances in the Experimental group would come from the holistic theory based on "the logic of children". Especailly important elements lead such success would be both that instruction was build on informal knowledge in students and intended to let them acquire meta-cognitive ability such as estimation by introducing the material to represent magnitude of ratio. Thus, it was suggested these two elements of the holistic theory are especially important for instructional intervention.

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