Articles

A Study about Calculations up to 100 in an Instructional Intervention Study[1]

YOSHIDA Hajime and SAWANO Kooji[2]

The purpose of the present study was to test effect of this kind of instructional intervention on understanding of simple addition and subtraction. One class of first graders in an public elementary school in Japan was given number representations based on 5 for two months. The class teacher always decomposed numbers over 5 as 5+x when he gave them. On the contray, none of other two first graders classes in the school decomposed numbers over 5 as 5+x during two months. For next four months, addition and subtraction with numbers below 20 were taught in the all classes. During these months, all classes taught addition and subtraction based on number system based on 10. Immediate and retention test were adminstrated. As a results, the experimental class showed superior performance to one in textbook class for both immediate and retention tests. However, compared to the correct percents, differences between two groups were bigger in strategies choice.

Key words : intervention, instruction, addition, subtraction

Many recent investigations have pointed out the importance of informal knowledge in the understanding of mathematical concepts by students (Fuson, 1992; Nunes, 1992; Saxe, 1988). Such researches also stressed that instruction should be based on the informal knowledge (Carpenter et al., 1993; Hiebert & Wearne, 1996; Mack, 1993). The present study is in line on such researches.

Many investigations made clear some factors of difficulties in addition and subtraction. One of main difficulties is that many curricula little reflect authentic experiences through everyday life of children (De Corte et al., 1996; Streefland, 1993). Other of difficulties is that many teachers pay much attention to procedure involved in a concept but less attention to meaning of the concept (Hiebert & Wearne, 1986; Resnick, 1983).

Almost all of interventional studies on addition and subtraction challenged to overcome these barriers in multidigit addition and subtraction (Carpenter et al., 1997; Fuson et al., 1997; Hiebert et al., 1996). Interventional programs in these studies intended to construct conceptual relation between the number words, written 2-digit marks, and quantities and to use these triads in solving multididit addition and subtraction. Because one of central conceptual structures in multididit addition and subtraction

1) This work was supported by a Grant-Aid for Scientific Research, Ministry of Education, Science, and Culture of Japan, No. 12610135, to the first author.
Correspondence concerning this article may be sent to YOSHIDA Hajime, Department of Psychology, Ritsumeikan University, Kyoto 603-8577, Japan (e-mail hyoshida@lt.ritsumei.ac.jp).
2) Hinokage Elementary School, Miyazaki

is ten-base number concepts, the concepts are used to represent numbers or to give word problems. However, none of previous studies suggested some difficulties in learning of simple addition and subtraction with single digit. Because many of first graders know count-on as well as count-all strategies in solving addition problems, it was believed that pupils are fairly competent in simple addition and subtraction with single digit.

None of researchers expected that there is a structure in simple numbers below 10. For example, Resnick (1983) divided the development of the understanding of number representation into three broad periods. The first, in preschoolers, is number representation. In this period children construct a representation of numbers that can be appropriately characterized as a mental number line. The second is found in the early primary school years. In this period, children can interpret numbers as compositions of other numbers. The third period appears in the later primary school years. In this period children's representation of number concept is modified to reflect knowledge of the decimal structure of the counting and notational system.

However, Yoshida & Kuriyama (1986) confirmed that for numbers below 10, preschoolers have number representation based on the number 5 rather than the number 10, not a simple mental number line indicated by Resnick. For example, in one of their experiments children were taught how to solve problems using the number 5 or the number 10. They found that children given the former solved almost all the problems correctly. In other experiment, children were asked to resolve numbers into 5's and x's or to find supplements to 10. It was significantly easier and faster for children to resolve numbers than to find supplements. Kuriyama & Yoshida (1988) demonstrated that this representation was not external but internal one.

In Japan, all national curriculum guidelines require teachers to teach numbers as ten-base concepts. All concepts related to numbers are taught as concepts reflected the decimal system. However, many of first graders have informal representational system based on the number 5 when they begin to learn addition and subtraction. Therefore, the present investigation tried to activate children's informal knowledge by presenting as composition of 5＋x in representing numbers over 5. As many previous investigations suggested, it would be expected that if instruction based on children ' s informal knowledge was given in simple addition and subtraction with single digit, pupils indicate superior understanding on numbers to ones taught according to a typical textbook. The purpose of the present study tested this hypothesis.

Method

Participants

The experimental group consisted of 31 first graders in a public elementary school of local city in Japan. The textbook group consisted of 31 first graders in the same school.

Tasks and procedure

Experimental instructional period: Mathematics lessons from the mid of May to the beginning of July were target in the present study. The subject matters taught were addition and subtraction. For this period, sums of addition problems were below 10. In subtraction problems both numbers of the problems were below 10. Contents of lessons for this period were "how many" (7 lessons), "increase or decrease" (7 ones), "addition" (7 ones), "subtraction" (9 ones).

Experimental group: In the Experimental group (E group) numbers over 5 were always represented as 5+x. For example, when teacher gave 8, 8 was shown as 5+3 as circles; 5 was in the upper line, and 3 in lower line. When the teacher wrote number on the blackboard or gave students worksheets, which included number, numbers over 5 were represented as 5+x.

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In teaching addition, addition based on the 5 was instructed if sums of problems were over 5. For example, consider 7+3. At first, 5 of 7 was represented in the upper line and remaining two of 7 did in the lower line. Then, the teacher added three circles on the same lower line. By counting all circles on both lines, the teacher taught the sum of 7+3. Other problems with sums over 5 were instructed in

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the same way.

In problems with sum of 5 or below it, the teacher did not mention these compositions at all. There was no difference between the experimental and textbook groups in these problems.

In teaching subtraction, instructional methods were basically same to ones in addition. Consider 9−7 as an example. At first the teacher wrote down five circles on the upper line and four ones on the lower line. Then, he removed five circles from the upper line and two ones from the lower one. By counting remaining circles, he taught subtraction.

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Textbook group: In the textbook group (T group), instructional ways on both addition and subtraction followed ten-base system. However, all problems used during the experimental intervention in the E group were ones below 10. So, class teacher of the T group did not give any instruction about numbers as composite of ten and unit. The T group taught 30 lessons in addition and subtraction for experimental instructional period according to the textbook.

In teaching addition, the class teacher followed a mathematics textbook. She arranged circles as number representation on a horizontal line when gave numbers to pupils. For example, let's consider 7+3. The teacher wrote seven circles on the blackboard, and then added three ones on the right side of the seven ones.

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In instructing subtraction the teacher used these representations in similar way. For a problem of 7−3, for example, she wrote seven circles on the blackboard, and then removed three from the circles.

Tests

Immediate test: The test was individually administrated one week after finishing lessons on addition and subtraction on the mid of July for both groups. Interviews were filmed. In the immediate test two kinds of tasks were included: arrangement of numbers and addition and subtraction. In the arrangement task, pupil was given numbers of 8, 12, and 16 and asked to represent these numbers on a sheet of paper. In computation task four problems were given to pupils: 4+6, 1+5, 7−5, 8−6. They were given marbles and asked to use them in solving problems.

Retention test: In the second semester, both groups taught according to the textbook, in which numbers based on ten-base system were shown. So, there were no differences between the E and T classes in concepts taught, main methods adopted, or main materials used because two teachers followed the same textbook and teacher's guidebook.

The second semester begins at September in Japanese elementary schools. Contents related to number and computation which were taught in the term were numbers to 20 (6 lessons), computation of three numbers (5 ones), addition (9 ones), subtraction (10 ones), word problems (2 ones), and addition and subtraction with 0 (2 ones).

The retention test examined concepts taught in the second term: addition and subtraction, finding supplement to 10, counting, and number arrangement. In addition and subtraction ten problems were used: 9+7, 6+8, 8+7, 9+4, 8+5, 16−5, 12−9, 11−4, 18−7, and 16−8. Five problems (2, 4, 6, 7, and 9) were used in finding supplement to 10. Pupils were given number (e.g., 6), and required to find supplement to 10. Counting task had six problems. Numbers used in the task were 14 and 17. These numbers were presented in the three ways: arrangement based on 10, one based on 5, and one at random. In the arrangement task, were given and pupils were asked to represent 8, 12, and 16 as circles on a sheet of paper, respectively.

Because of the school's schedule, we had no time for individual interview and test. So, retention test was collectively given on the mid of December in each group. In order to avoid "ceiling effect", time limitations for all tasks except number arrangement were set. 40 seconds in the addition and subtraction seconds were given to students, 10 ones in the supplement task, and 40 ones in the counting task.

Results

Immediate test

Correct percentages in the arrangement task were 94.7% in the E group and 92.7% in the T one. There was no significant difference between two groups.

However, there were clear differences in strategies utilized between the both groups. By analysing video film, we found main two strategies in the arrangement task.

D strategy: One arranged based on 5. Pupils arranged five circles in the upper line and the remaining number in the lower line.

L strategy: Pupils arranged circles as number designated by the problem in a line.

Figure 1 shows percentages in the D and L strategies used for both groups. The E group depended upon the D strategy while the T group did upon the L. These results would be natural because pupils in both groups used strategies learned in their classroom, respectively.

Figure 2 indicates correct percentages in addition and subtraction problems for both groups. There were significant differences between the both groups in addition, $\chi^2(1)=6.98$, $p<.05$, and in subtraction, $\chi^2(1)=5.21$, $p<.05$.

There were also differences between the strategies used in the two groups. Main two strategies in addition were D and L. In D strategy, pupil decomposed number over 5 into 5 (upper line) + x (lower one), and added another number to the lower line. In L, pupil arranged number in a line at first and added another number on the right or left side of the line.

In subtraction, similar strategies (DW and E) were observed. In the DW strategy, pupils dealt with 5 as a total. They decomposed number over 5 to 5+x, and subtracted these five marbles simultaneously. Consider 8−6. Pupils decomposed 8 into 5 (upper line) + 3 (lower line), subtracted all 5 on the upper line and 1 from the lower one, and responded two. As other type in the D strategy, pupil subtracted 3 from the upper line and 3 from the lower one. We classified the type into D because pupil treated 5 as a total. In another type of D, pupil arranged 8 marbles in a line and subtracted 6 marbles simultaneously. We classified this type into D because six marbles were dealt with as a total. However, in the E strategy pupil sub-

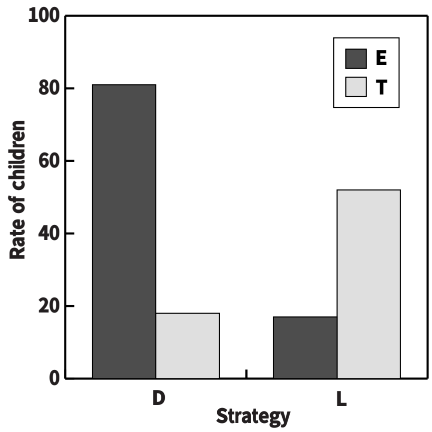

Fig.1. Strategies in both groups for the arrangement task: Immediate

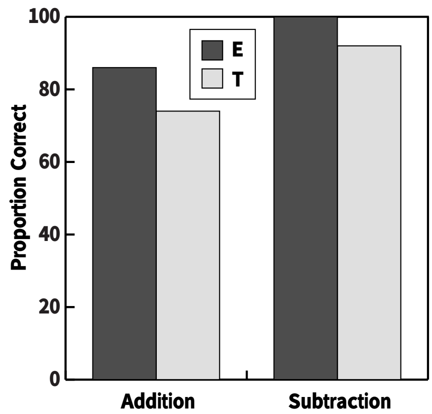

Fig.2. Correct percentages in both groups for addition and subtraction: Immediate

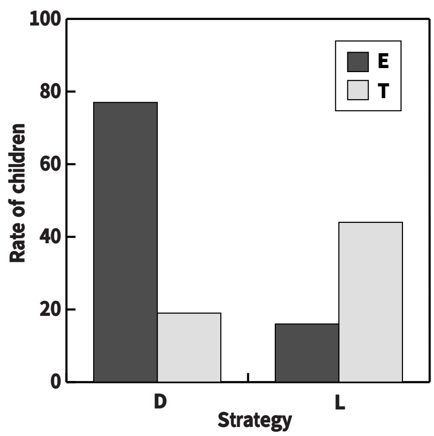

Fig.3. Percentages of strategies used in addition: Immediate

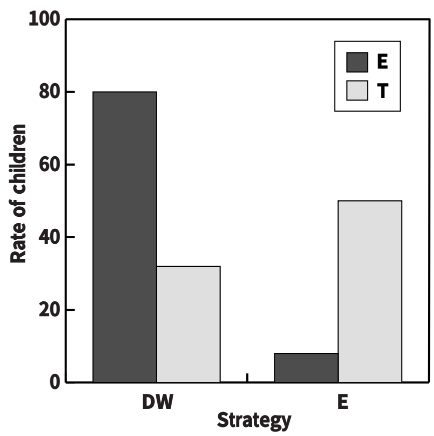

Fig.4. Percentages of strategies used in subtraction: Immediate

tracted one by one. In the problem (8−6), pupil subtracted one by one from 8 in a line or 5+3 in two lines.

Figure 3 shows percentages of each strategy utilized in both groups for addition. There were significant differences between strategies used in the two groups for D, $\chi^2(1)=34.13$, $p<.01$, and for L, $\chi^2(1)=8.95$, $p<.01$. Amounts of differences between the E and T groups in strategies were highly bigger than differences between these groups in percent correct shown in Figure 2.

Similar tendencies were found in subtraction. Figure 4 indicates percentages of each strategy utilized in both groups for subtraction. There were also significant differences between the two groups in DW, $\chi^2(1)=19.98$, $p<.01$, and in E, $\chi^2(1)=10.33$, $p<.01$. These results also indicated that the difference of percentages of strategy utilized in both groups were fairly larger compared differences of correct percentages shown in Figure 2.

Retention test

Because the retention test was administrated collectively, it was hard for us to analyze strategies which pupil used except the arrangement task. Correct percentages of addition and subtraction task were shown in Figure 5. Subtraction was divided into problems without borrowing and with one. While there was no difference between the two groups in addition, there were significant differences between the groups in subtraction without borrowing, $\chi^2(1)=8.54$, $p<.01$, and with borrowing, $\chi^2(1)=11.42$, $p<.01$.

In the second semester, no of class teachers in both groups taught any ways on addition and subtraction based on 5 during five months after Immediate test. Both teachers followed the same textbook, in which numbers are instructed by ten-base system. However, the E group demonstrated superior performance in subtraction to the T group.

Because we set time limitation, some of pupils did not answer some problems at all. So, we examined percentages on problems that pupils did not answer at all for both groups. Figure 6 indicates such non-response percentages in addition and subtraction for both groups. While there was no difference between the two groups in addition, the T group showed non-response percentage higher than one in

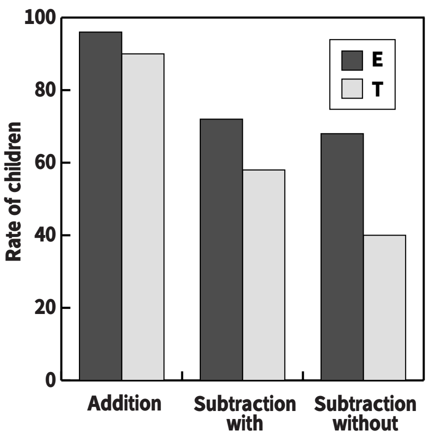

Fig.5. Correct percentages in both groups for addition and subtraction: Retention

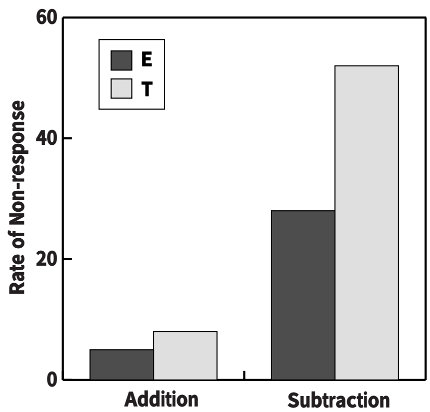

Fig.6. Rate of non-response in both groups for addition and subtraction: Retention

the E group, $\chi^2(1)=15.76$, $p<.01$. Thus, the E group solved problems more and did more exactly than the T group.

Figure 7 indicates correct and non-response percentages in the supplement task for the two groups. There were significant differences between the two groups in the correct percentage, $\chi^2(1)=28.09$, $p<.01$, and in non-response percentage, $\chi^2(1)=28.10$, $p<.01$. Ways of instruction on finding supplement to 10 for the second semester were basically same for the two groups because they followed the same textbook. Although there were no differences in instruction, the E group demonstrated superior performance in the supplement task.

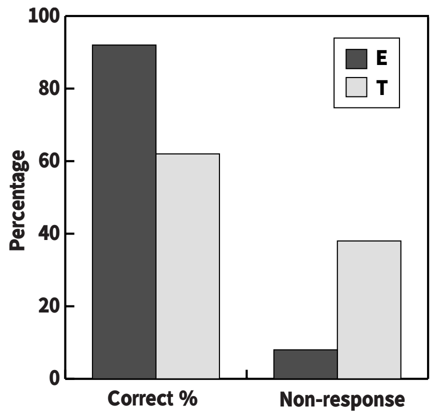

Fig.7. Correct percentage and rate of non-response in the task of finding a supplement: Retention

The counting task were presented in the three ways; one based on 5, one based on 10, and one at random. Figure 8 shows percent corrects in each arrangement for the two groups. There were significant differences between the groups in arrangement based on 5, $\chi^2(1)=7.89$, $p<.01$, in one based on 10, $\chi^2(1)=8.94$, $p<.01$, and in random, $\chi^2(1)=12.95$, $p<.01$.

Correct percentages in arrangement task for the two groups were very similar, 99% in the E and 96% in the T groups. There was no difference in the correct percentages for the two groups. However, we were able to analyse pupil's strategies in this task because they wrote circles on a test sheet of paper. Main five strategies in the task were found; D, 10, L, half, and random strategies. In D one, numbers were represented based on 5. In 10, 10 circles were represented in the upper line and the remaining numbers did in the lower line. In L, all numbers were represented in a line. In half, half of numbers given in the problems were represented on the upper line and the other half did

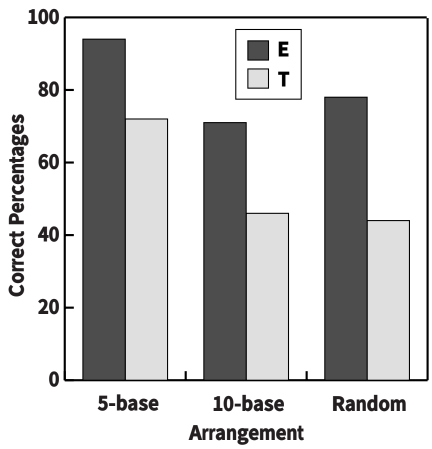

Fig.8. Correct percentages in both groups for the counting task: Retention

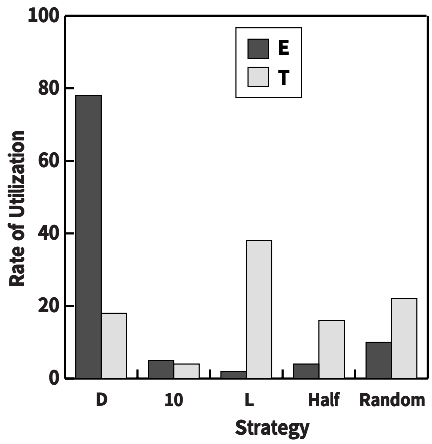

Fig.9. Utilization rate of strategies used in the arrangement task: Retention

on the lower line. Figure 9 shows percentages of strategies utilized for the two groups. There were significant differences percentages between the groups in all strategies except 10.

Discussion

The purpose of the study was to test the hypothesis that instruction based on children's informal knowledge was given in simple addition and subtraction with single digit, pupils indicate superior understanding on numbers to ones taught according to a typical textbook. The obtained results confirmed this hypothesis clearly.

Instructional intervention based on informal knowledge in children was given to first graders for about two months in the present study. As results, first graders demonstrated deeper understanding in many aspects of numbers even in the Retention test after five months as well as in the Immediate test. The E group was given representation based on 5+x in numbers over 5. But in the T group, numbers just represented without such structure. Pupils were shown numbers in a line. In this representation there was no structure because numbers used were ones below 10. So, it would be natural that the E group utilized strategy relied upon 5 more than the T group in the Immediate test. However, superiority of the E group was confirmed in the Retention test on about five months after finishing experimental instruction.

Thus, the present investigation demonstrated that instruction based on children's informal knowledge was successful in their learning. However, it should be considered that such superiority found in the present study might be limited to learning of simple addition and subtraction task with numbers below 10. In task with numbers over 10, numbers are shown based on decimal number system. Previous studies already indicated successful instructional intervention in multidigit addition and subtraction (Carpenter et al., 1997; Fuson et al., 1997; Hiebert et al., 1996).

Because children have rich informal knowledge on simple numbers below 10 or simple addition and subtraction, it may be considered that first graders in elementary school have no any difficulty in simple addition and subtraction with single digit. If so, we may not expect improvement in performances in such addition and subtraction even if given experimental intervention for the task. If this was true for all pupils in first graders or kindergarten children, research and development on instructional ways based on informal knowledge in children would not be needed. However, we know some pupils have a great difficulty even in learning such a very simple kind of task. Therefore, intervention based on 5 would be suitable be appropriate for such pupils.

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(Accepted Dec.18, 2001)