Public Lecture

What is Understanding[1)]

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1. Some of the challenges facing academic achievement

In this essay, I am going to analyze the process of "understanding, or comprehension" from a psychological perspective. "To understand" is in itself learning, and the process of understanding by learning should be an essentially fun process.

In present day Japanese society, we find various social problems concerning the school situation where learning is directly involved. One such challenge is the falling level of academic scores. This problem is seen not only among primary school students but also among college students. One recent shocking case involved a student of a famous university who was unable to calculate fractions. As a consequence, people are increasingly concerned about the on-going decline in academic achievement levels.

Now let us consider the global position of the academic standard of Japanese children. There is an academic achievement survey known as IEA (Nagasaki,1997) in which40 countries around the world participate. This survey involves a tremendous number of children, and can therefore only be implemented every 5 or 10 years. The test assesses abilities in math and science, and in the past two tests Japanese students have achieved the highest scores. This result is widely known around the world and it is commonly known that Japanese children have high academic standards. However, in the most recent test, children in Singapore and South Korea outperformed Japanese children. It is fair to say, the academic ability of Japanese children is now facing a decline.

Studies have revealed that the most serious issue facing our children is their motivation to learning. In studies on motivation and willingness to learn, such as "I am interested in studying" or "I want to learn more", Japan ranked 39th out of 40 countries. So we can see that even though the level of academic performance is among one of the top, Japanese children are the least motivated towards learning. This is a serious problem. If we continue to allow such a situation to occur, it may lead to a serious break down in scientific technology, not to mention general education and academic levels. This is one of the examples of how society is interested in the issue of learning.

These kinds of social concerns are always present when it comes to "understanding" or

1) This paper was based on the Ritsumeikan Saturday Public Lecture Series (2559th, June 2, 2001), and originally published Ritsumeikan Journal of Human Sciences, No. 2, 121-131 (2001).

"learning," nevertheless it is not my purpose to discuss such social concerns in this paper. The purpose of this paper is to think about what is learning, in other words to examine the meaning of learning. While some children find the process of learning easy, others do not. Why is learning such a hard task for some?

Several possible reasons have been proposed as to why learning is difficult for some. The media also often describe various reasons behind difficulties associated with learning. One of the most discussed reasons is the issue of curricula in which the contents of teaching is continually reduced. Under the latest revision by the Ministry of Education, Culture, Sports, Science and Technology (MEXT), the curriculum will be cut by 30%. Since we are experiencing a reduction in academic competency even now, much controversy exists over what will happen to our children's academic standard with 30% less content. Another issue is the matter of class size. The average number of Japanese pupils in one class is 32～33, about double that of the USA and Europe. In other words, Japanese teachers are forced to teach a larger number of students compared to American and European counterparts. The issues on which the media focus are mainly due to external factors. Of course, such external factors are very important issues when it comes to observing the difficulties that some children have with learning. However, as a psychologist, I am going to examine why learning becomes difficult for children by trying to understand what is actually going on inside children's minds.

When learning advances, "knowledge" increases inside children's minds. By progressing from first, second, third grade, etc., children continuously learn various things. However, what is actually happening inside their minds, or to put in another way, what happens to knowledge inside the mind when learning advances. Concerning this matter, when asked "what is happening inside children's minds?" many people respond "knowledge accumulates little by little." This view is just like water filling a bucket, as we continue to pour water into a bucket, the water level rises. The common view is that the accumulation of knowledge works in the same way.

2. Behaviorism and learning

This approach is closely related to psychological theory. Also it relates to how adults and teachers teach students who do not understand. How can we guide struggling students based on the position of knowledge accumulation?

Psychology was established as an academic discipline at the end of 19^{th} century. Wilhelm Wundt founded a psychology lab in the philosophy department of Leipzig University in a small town in Leipzig, Germany. It is marked as the time when psychology was established as an academic field. Over 100 years have passed since psychology was recognized as a separate discipline, and from the start until the 1960's, the main stream of psychology was so called behaviorism, where psychology was understood as an academic field to study people's behavior. The basic framework for psychology since its foundation in the 19th century has been to study the relationship or connection between a stimulus, which precedes certain behavior, and behavior. That is to say, if one tries to study a certain behavior, one should study the relationship between a preceding stimulus and the behavior. This was how psychological study was conducted until the end of the 1960's (Yoshida, 2000).

Given such a stand point, how should we instruct children? Let me give you an example. A university student, an acquaintance of mine, was tutoring a ninth grader. One day he popped into my office and asked, "When asked what is 13-3, my student answers 0. What can I do?" The ninth grader was answering 13-3=0. When you have such a child around you, how can you, as an adult, instruct and teach him/her? Many people would draw 13 circles on a piece of paper, take 3 circles away, and then have the student count the remaining circles. This method is quite straight forward and there is nothing wrong with it. However let me remind you that this student is in the ninth grade. This cannot have been the very first time that he/she encountered this method. By the time he/she reached the ninth grade, teachers, parents or even brothers and sisters must have taught him/her many times using a similar method. Nevertheless, he/she still answers 13-3=0, so it is clear that this method has not been useful for this child. The question then is how should we help, or instruct him/her?

In such cases, many people start with easier problems, such as 5-2, 7-3, or try to explain using easier and simpler language. It is unlikely that there is a person taking a different approach. When we analyze this case from a psychological perspective, switching to an easier problem or repeating the explanation means the teacher is giving the child a stimulus. The methods stated here are approaches that change the stimuli that are presented to a child when the child does not understand. No matter how hard the teacher tries to explain, not understanding is the child's response. As a consequence, the teacher starts to perceive the child as being "stupid or dull," and therefore "it is useless to teach him/her," and eventually the teacher may give up trying to instruct him/her. Such a child then becomes a visitor in the classroom, and in fact we find quite a few pupils of this nature in classrooms.

What is wrong with such instruction or approach? We need a new approach in tackling such a problem.

3. What is cognitive psychology?

An innovative approach was introduced in the 1970's from a new psychological theory called cognitive psychology. Since its introduction, cognitive psychology has had a significant impact, and as a result research based on behaviorism has diminished. In this new school of psychology, research focuses on what is going on inside the mind. In this approach, the relationship between stimuli and behavior is no longer as important as in behaviorism. The most important research theme is to understand and study what is happening inside the mind.

In behaviorism, this inner process received little attention. However, without stepping into the inner process, the very core of human beings would remain undiscovered for ever. Thirty years have passed since the cognitive approach became main stream. By taking this approach, we can gain an insight into the inner part of human beings, because here psychologists try to study what is going on in people's minds. Since the introduction of this new school of psychology, various innovative pieces of research, which were impossible under the framework of behaviorism, have emerged.

Now, let's examine the learning process of children within the framework of cognitive psychology. Naturally, various things are taught to children in the learning situation. If information taught to children is taken into their minds as it is presented, in other words, if children understand word by word the instructions of teachers and parents, teaching or educating would be a relatively easy process. We as instructors would not have had to work hard in university to qualify as teachers. So, is given information literally taken into children's mind as it is? The following is an anecdote from a student of an acquaintance of mine when the student was in the first grade. A teacher taught children how to sow seeds in their science class. They were sowing seeds in pots, and when it came time to instruct the students where to place the seed in the soil, the teacher told the children to place the seed about as deep as the length of their thumb. Later, the pupils were given a quiz on sowing seeds in which they were asked, "What is the correct depth to place a seed? The pupils were asked to select one of the following answers; on top of the soil, just under the surface of the soil, or deeply in the soil. The child in question incorrectly chose the last answer. When his/her parents asked why he/she made such a stupid mistake, he/she answered "my teacher told me to 'get my thumb out and place the seed around here' so, I put my thumb on the test sheet." This example illustrates how the child took the instruction word for word, but this is a rare case.

As study continued to focus on the inner processes of people, how people think, and how knowledge is structured, researchers reached the following conclusions. People retain some previously learnt knowledge that is referred to as acquired knowledge. When one encounters and tries to understand new information, this new information and acquired knowledge interact. Most of the time this interaction takes place unconsciously. Unknowingly, we draw on some of our acquired knowledge. By doing so, we comprehend or infer the information. People do not simply add the new information to their knowledge, but rather the new information interacts unconsciously with our acquired knowledge.

In such a case, what kinds of interactions occur? Sometimes, the acquired knowledge functions as a promoter (Carpenter, 1986). However, it can also work as an inhibitor, blocking the comprehension process of the new information (Leinhards, 1988). If the interaction with our acquired knowledge is positive, comprehension is promoted, whereas, if the interaction is negative, comprehension is blocked. To really understand this function, it is necessary to actually have people experience it. So let me give you a simple example. I want you to read the following passage. Please think about what the sentences are describing.

> The procedure is fairly easy. First of all, divide the objects into several mounds. Of course depending on the amount it is possible to make just one mound. If you do not find the necessary thing there, you need to go to another place, but otherwise, you are done with the preparation. The key is not to try to do too much at once. It is better to do too few at once than too many. You may not realize the importance of this advice immediately. However, it may result in trouble or cost you more if you do not follow this advice. It may seem a complicated process at first, but soon it will become an ordinary part of your daily life. At least for the time being, it is hard to imagine that this task will become unnecessary nor anybody can predict so. When everything is completed according to the procedure, organize by dividing the objects into several mounds. Then place them in their assigned places. Soon, they will be used again, and the process will be repeated. It is troublesome, but nevertheless it is a necessary part of our daily lives.

You probably understood every sentence, however it is unlikely you understood what the passage as a whole was talking about. This passage is about "laundry". If you read this passage again with this notion in mind, I imagine you will be able to connect the sentences and comprehend the entire passage.

This simple experiment suggests that when we receive new information, it does not make sense unless it interacts with our acquired knowledge. If it does not make sense, we cannot understand or find any meaning in it. I hope this kind of thing does not happen in the classroom, however as the falling academic achievement and poor performance indicates, unfortunately it maybe happening quite often in the classroom. You can easily imagine the situation in which a child is placed when he/she sits for 5 or 6 hours in a classroom without understanding what is going on. For such children, school is nothing but torture and no fun at all.

Let me give you another example. Please read the next passage. If you think you have understood the passage, please recall what was written without rereading the passage.

> The captain must have been dead for quite a while. He was counting his subordinates as they came back. After the dogfight, the

Japanese planes returned to their base in threes and fours. He was shot in the chest by the enemy and later they found that it was a fatal wound. He arrived at the headquarters and reported to the commander. However, immediately after arriving, he collapsed on the floor. Getting off his plane, the captain stood on the ground and looked into the sky through his binoculars. A captain was on board among one of the first planes returned. When the corpse was checked, it was already cold. Nevertheless, the captain's body was as cold as ice. After making sure that the last plane had returned, he wrote a report and headed to headquarters. The surrounding officers hurried to help him, however, he was already dead. His soul had made the report. A body that was alive moments ago could not have become so cold. His face was pale but he was strong. It must have been the dead captain's strong sense of responsibility that made this miracle happen.

Most people can recall this passage by switching the order of the sentences to make sense of it. Without any instructions, the reader voluntarily changes the order of the sentences when taking in the new information and interpreting the meaning to deepen their understanding. Very few people recall these sentences as they were written, "The captain must have been dead for quite a while. He was counting his subordinates as they came back."

Let's examine what this simple experiment indicates. In this example, the readers' acquired knowledge is activated and by vigorous interaction between it and the sentences, readers are able to gain a deeper understanding. So what kind of acquired knowledge was utilized? We have an acquired knowledge that a story is structured in four parts, namely, the introduction, development, twist and conclusion. When we read a segmented and shuffled story, we activate our acquired knowledge of story structure and try to derive a deeper understanding. When we have this kind of interaction, we can understand far better than simply the level of information provided. If the mechanism of learning works in this way, it is quite ideal for children, however unfortunately, we rarely see such cases in general learning. Like the first example of laundry, in some children the new information and the acquired knowledge do not interact with each other. Therefore, some pupils are left out. A similar situation is found in the learning environment in schools.

As these cases demonstrate, learning something is a process of interaction between old and new knowledge in the mind, and not a process of accumulating new information in the mind. Through interaction between knowledge groups, knowledge is reorganized in a new way. That is to say, knowledge is restructured, the mind reorganized, and acquired knowledge updated. This is the process of learning.

4. When informal knowledge precedes formal knowledge: the development of counting, addition and subtraction.

Children are taught in schools. Let us examine what happens in the children's learning process. Children start school in April. If we ask a first grader, "what kind of place is school?" all children respond "School is a place to study. Kindergarten is a place to play." All children make a clear distinction between kindergarten and school. Currently, they do not have sociology or science classes, so their main focus is arithmetic and Japanese language. In April and May, students learn addition and subtraction. They are taught tasks like, "3+6 equals? Or 6-3 equals?" This kind of problem is quite easy for first graders. Children enter school expecting to study, and when they find class is easy, they enhance feelings such as "school is easy and learning is fun, so I want to go to school." Because they can understand, studying becomes fun. So they willingly go to school. This is extremely important from the perspective of learning and motivation.

We must question why such addition and subtraction taught in April and May are easy for first graders. What makes these tasks easy for children? Common answers to such questions are, "they can count using their fingers," "the problems deal with small numbers," or "they already knew the problems." It is important to recognize that these answers are given by teachers and adults. For instance, when we deal with a ninth grader who answers 13-3=0, and try to figure out a way to teach him/her, if we know what makes the problem easy, we can find a way to teach that student based on that knowledge. There have been a number of findings on this matter in recent studies. Upon learning new things, people achieve various aspects of psychological development. In other words, various cognitive abilities are developed inside people's mind. When such development reaches a certain level, certain problems and concepts become easy (Yoshida, 1991).

Let's look at another question. How can we teach a child when he/she does not know the answer to 3+6? Most probably people will draw circles and make the child count them to teach the child 3+6=9. If the child is able to understand using this method, that is fine, however, if he/she still does not understand, what can we do?

Before figuring out the answer to this question, let us examine how the inner development of people is the base for resolving problems. In theory, when a child has developed functions relating to addition and subtraction, they can add or subtract as a result. For instance, it is related to the ability to count out loud, "1, 2, 3, 4, 5, 6" when there are 6 objects. This kind of counting aloud is a very easy task for a child over a certain age. However, even a simple task has to go through several developmental stages. Let me explain using the counting mechanism. The first stage is mechanical memorization.

Usually, a child can memorize numbers around the age of two. A child who has memorized numbers can easily say "1, 2, 3, 4, 5, 6". However, this is simply a memorized response, and he/she cannot use these numbers as a tool for thinking, inference, or recognition. This situation is just like the case in which a person sings in a foreign language, chanson or canzone, without knowing the language. When he/she stops in the middle of the song, he/she can not pick up singing from where he/she stopped. He/she needs to start from the beginning again. These situations are very much alike. The child at the beginning stage, has simply mechanically memorized the numbers, so he/she cannot pick up counting from where he/she has stopped. He/she always needs to start from 1. In the next stage, a child learns to differentiate one number from another. He/she successfully recognizes numbers separately like "1 and 2" "2 and 3" "3 and 4" and so on. Once he/she can understand those numbers are separate entities, they can identify differences in numbers, and form one-to-one correlations between numbers and objects. "One-to-one correspondence" means to allocate the number 1 to an object. However, a child at this stage cannot stop counting when told to count from 1 to 8. He/she cannot stop in the middle and keeps counting until he/she reaches the end of his/her knowledge. We often find a good example of this in the classroom of grade school in April or May, when a teacher draws circles to help children count. When counting up to 6, most children stop counting at 6 when the teacher stops at 6, but some children keep going to "7, 8, 9". Some teachers may warn children not to play around, but in fact some children are unable to stop.

In the next stage of development, a child is able to count from 1 to a certain designated number. That is, when he/she is told to count from 1 to 7, or 1 to 13, he/she can do so. It usually happens when a child is in his/her late 3's or early 4's. Once a child reaches this stage he/she is also able to master addition as a result. It is commonly understood that people master addition naturally at a certain age, and it appears to be a natural process, however one masters addition because such counting ability has developed inside the child's mind. Furthermore, such counting ability continues to develop into further stages. Clearly, it is necessary for a child to add the ability to be able to stop counting aloud. If one cannot stop counting, he/she cannot master addition.

In the next stage, a child is able to start counting from a certain number. A child can follow instructions like, "count from 6 to 13." In this stage, a child can add numbers using a different mechanism, known as "count-on" (counting all elements is called "count-all"), in addition to the method of counting all available elements. This usually happens at about the age of 5. "Count-on" is an extremely complex mechanism, and consists of three steps. First, he/she identifies the larger number of the two numbers presented. In the case of 3+6, he/she identifies 6 is larger. Second, he/she remembers the larger number. When we observe children of this age doing addition, they often say 6 aloud, this action refers to this step. Finally, they add the smaller number to the larger one. The method of "count-on" is useful in many ways. One of its characteristics is the lower number of times it is necessary to count, and as a result he/she can add quickly. On average, children learn this skill in their 5's.

Now, let's return to the previous question. "Why is addition taught at the beginning of the first grade easy for children of that age?" It is easy because about half of all children have mastered the method of "count-on" by the time they start school. If children learn this method before starting school, who do they learn it from? Do parents teach children this method? Or do they learn this method in kindergarten? It is unlikely parents or kindergarten teachers teach preschool children this complex method. In fact, children are not taught this method, they figure it out for themselves. Children of this developmental stage start going to school. The method children are taught at the beginning of the first grade is as follows; if the question is 3+6, write "3" on the left plate and "6" on the right plate, and add both numbers on the plates by counting all numbers aloud. This is the "count-all" system that I have mentioned above.

Therefore for children, the addition taught at the beginning of school education, is in fact a method they have already mastered. Consequently, the addition taught in school is very easy for them. For children, the addition taught in the first grade, is knowledge they have already acquired, therefore children's knowledge is more advanced than the new information (Yoshida, 1991).

5. When formal knowledge precedes acquired knowledge: the study of fractions

I have just described a case where children's knowledge is more advanced than formal knowledge. However, we seldom see such cases. Overwhelmingly, the knowledge taught at school is more advanced than children's own acquired knowledge. What happens to the learning process in such cases?

I would like to examine the mechanism of calculating fractions. The media often talk about this as an area that children often fail to understand. There are high school students or even college students who cannot calculate fractions. What's more, in the new revision of the educational curriculum, fractions will be one of the most drastically reduced areas. So fractions are widely talked about in society, and one of the reasons for that is the complexity that the concept of fractions involves. It will bore you if I go on talking about such complexity, so I will focus on just one complex aspect. The concept of fractions is taught in primary school from third grade to sixth grade, so children are taught for four years. No other concept is taught for such a long period in grade school. There are several basic notions related to understanding fractions, and in this paper I would like to examine the notion of equivalence or the notion of large and small. In other words, the notion of which fraction is larger or smaller, or which fractions are equal. This is so basic that it is taught in the first class of the first year in which fractions are introduced. No matter how basic this concept is, it is known that it is fairly difficult to understand. Which numeral is larger, $\frac{2}{3}$ or $\frac{2}{5}$? Or are $\frac{2}{3}$ and $\frac{4}{6}$ equal or not equal? By asking such questions and getting answers, we can see how each child has conceptualized the notion. We can understand how a child has understood correctly when he/she gives the correct answer using the right method. But we can also analyze how he/she has misunderstood the notion and what kind of knowledge he/she has from his/her incorrect answer.

Let us examine the background of mistakes. Some of the general reasons for mistakes are; a child answers haphazardly when he/she does not know how to solve the problem, inattention or careless mistakes and so on. For example, simply mistaking addition and subtraction due to lack of attention, or recognizing the number 4 as the number 1. There are also cases involving blind or unreasonable mistakes. However, there may also be cases that are completely different from the ones mentioned above. It is in such cases that a child's knowledge and thinking ability is reflected in the mistake itself (Yoshida・Kuriyama, 1991). This type of mistake is expressed via a consistent method inside his/her mind. When given different problems, the result is different, however the basic mechanism remains consistent. I call this mechanism, an "error mechanism." If we can understand this mechanism in detail, we will eventually be able to understand the child better.

So let us analyze this "error mechanism" using the example of fractions. Let's use the problem of arranging the following fractions in order of largest to smallest: $\frac{2}{5}$, $\frac{2}{3}$, $\frac{2}{7}$.

One of the most common mistakes is,

$$\frac{2}{5} < \frac{2}{3} < \frac{2}{7}$$

However, in the case of $\frac{4}{7}$, $\frac{2}{7}$, $\frac{6}{7}$, the same children answer

$$\frac{2}{7} < \frac{4}{7} < \frac{6}{7}$$

Thus they answer the second problem correctly, but the first problem incorrectly. A child making this type of mistake is identifying the same number, whether it is the denominator or numerator, and focusing on the different numbers, then rearranging the numbers in order of largeness. In the mind of such children, in the first problem, since the numerators are the same, the larger the denominator, the larger the number. In the second problem, since the denominators are the same, the larger the numerator, the larger the number. So even though he/she gets the answer right in the second problem, it is not correct in the true sense. Children who make this type of mistake are using their knowledge of whole numbers to solve fractional problems, and we find many children who make this type of mistake.

Let me introduce a different type of mistake. As above, the tasks require arranging the fractions in order. Let's say a child answers as follows:

$$\frac{2}{7} < \frac{2}{5} < \frac{2}{3}$$

$$\frac{6}{7} < \frac{4}{7} < \frac{2}{7}$$

What kind of understanding results in such answers? In this case the child has partially understood the basic notion of fractions, namely the parts and the whole. To summarize, the concept of fractions (although this is not limited to fractions) is to divide the whole into equal parts, and as we divide the whole into more parts, the smaller each part becomes. Therefore the number of parts and the part's size have an inverse relationship. Children who answer as above do not fully understand this inverse relationship, but they apply their acquired knowledge to only one of the parts, the denominator or numerator, of the fraction and not to the fraction as a whole. Thus, compared to children who only use their knowledge of whole numbers, we can say that children who make this kind of mistake understand fractions imperfectly.

Now, let us see how children acquire the concept of fractions starting in the third grade.
Figure 1 shows a four year follow-up study of how one child comprehends the notion of fractions.

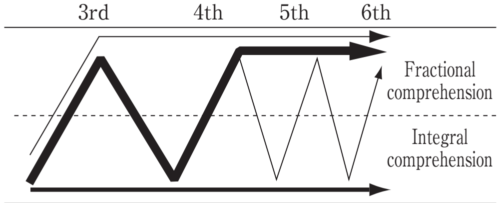

Fig 1 Developmental transition of comprehending the largeness of fractions

Naturally, a child only has knowledge of whole numbers when he/she starts learning about fractions. About 10% of all children can understand the largeness of fractions using knowledge of fractions. In other words, about 10% of children thoroughly understand fractions in the third grade. About 50% of children seem to understand fractions when they are taught in the third grade. However there is a one year blank before they pick up fractions again in the fourth grade, and many children tend to forget about fractions and only remember the notion of whole numbers. In addition, about 20% of children consistently try to understand fractions using their acquired knowledge of whole numbers even though they are taught fractions in the 3rd, 4th, and 5th grades.

Lastly, I want to examine how such an error mechanism appears as the outcome. The error mechanism, as I mentioned above, often appears differently in every child's answer. Therefore it is impossible for most teachers to identify the mechanism. Now, what are the children's actual responses? Please examine the following equations.

(1) $2\frac{5}{6} + \frac{10}{12} = 2\frac{8}{12}$, $2\frac{3}{8} - \frac{5}{6} = \frac{5}{24}$

(2) $2\frac{3}{11} - \frac{9}{11} = 1\frac{4}{11}$, $3\frac{3}{4} + \frac{1}{3} = 4\frac{3}{12}$

These are answers given by fifth graders. Please try to estimate what kind of error mechanism the child applied. On line (1) above, two answers are given. The same child solved both problems, therefore, the left and right problems must have been solved using the same mechanism. If you estimate a certain error mechanism and it works for the left problem and not for the right problem, it means the estimated error mechanism is not the one applied. Please try to estimate the error mechanism used for the problems on line (2). Were you able to easily estimate the applied error mechanism? I assume it was quite difficult to estimate the one child used. Let me explain. A child solving the problem on the first line knows how to convert mixed fractions to improper fractions. Therefore, he/she multiplies the whole number 2 by the denominator 6 and gets 12. He/she can also reduce the fractions to a common denominator. So when he/she converts $\frac{5}{6}$ to a common denominator 12, it becomes $\frac{10}{12}$. Then he/she adds 12 (the result of 2×6 to change it into an improper fraction) and 10 (used to reduce it to a common denominator), and gets $\frac{22}{12}$. As a result, $\frac{22}{12}+\frac{10}{12}$ becomes $\frac{32}{12}$ before being converted into a mixed fraction $2\frac{8}{12}$. I will leave the second problem to your own estimation.

When teachers encounter an incorrect answer, is there any teacher who instantaneously understands the process of the child's error mechanism? The answer is "no". Even a teacher who deals with students every day cannot see the mechanism and the process of the mistakes. If the teacher can estimate the error mechanism and mark the test using the child's mechanism, the correctness from the child's perspective, will increase dramatically. If the grade were 15 points when marked in the normal way, it would become 70 or 80 when marked using the child's own standard. If teachers and adults are able to estimate children's error mechanisms, what implication would it have on the teachers and adults themselves? It will go beyond the framework of teaching skills. To put it in an exaggerated way, it may influence them significantly and result in them changing their philosophy of children. As for teachers, it may result in a drastic change of their actual teaching manner. Such efforts for change take shape in the reviewing of the curricula and the implementation of research into curriculums based on "subject logic" that is coherent with "children's logic" (Yoshida & Sawano, 2001)

A considerable number of children who possess such error mechanisms are present in our schools. I have closely studied such children and summarized my observations in the following chart.

This chart classifies children according to their calculation ability. Children in group 5 exhibit the highest ability, while those in

Chart 1 Ratio of the number of children with error mechanisms & the mean

Ability group	Ratio of children	Ave.no.of error mechanisms per child
5 (high)	2.8% (3/108)	0.03
4	16.0 (20/126)	0.19
3	50.9 (45/89)	0.68
2	80.6 (42/52)	1.32
1 (low)	87.0 (34/39)	1.94

group 1 exhibit the lowest ability. According to this chart, only about 3% of children with the highest ability have consistent error mechanisms. Of the approximately 50 % of average children found in group 3, about half of them, have error mechanisms. Whereas, 80 ～90% of children in the below average groups have error mechanisms. On top of that the matter would be simple if one child only had one error mechanism, but the reality is not so simple. When we analyzed what kinds of error mechanisms occur in each child, the results were as follows. Children in the average ability group had one or fewer error mechanisms, while children in the lower than average range had between 1 and 2, or as many as 7 error mechanisms. If we studied their mechanism and graded tests from their point of view, most of them would get closed to 100. So children solve problems using mechanisms they think are correct. However, the returned papers are mostly marked incorrect. They are examples of children making unthinkable mistakes from teachers' points of view, and as a result a perfect divide is formed between teachers and children. Consequently, understanding children from inside becomes an impossible task. Therefore, I want to conclude by saying that it is necessary to take into account many facets involved in children's learning.

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